Comments on "Dissipation and Scattering Matrices of Lossy Junctions"

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In the above short paper,[1] some errors should be corrected and some remarks seem to be of interest.

Equations (5), (25), and (32) are not correct; the correct equations are

$$q_{12} = -S_{11}^*S_{12} - S_{11}S_{13}^* - S_{12}^*S_{13} \qquad (5)$$

$$y_{+1} = \coth\,(\alpha_{+1}l + j\Theta_{+1}'/2) \qquad (25)$$

$$\tan\,(\Theta_{+1}/2) \approx 2\delta_{+1}Q_{+1}. \qquad (32)$$

Furthermore, the matrix Q is, in general, not positive real provided $|q_{12}| < q_{11}$ (see sec. II, eq. (7)). We will prove this with the aid of a counterexample: for the matrix S with the elements $S_{11} = 0.1$, $S_{12} = S_{13} = 0.5$ (we use the same notation as in the Helszajn paper), we have $q_{11} = 0.49$, $q_{12} = q_{13} = -0.35$. Consequently, the inequality $|q_{12}| < q_{11}$ is satisfied, but, as it can be easily verified, this matrix Q is not positive real: the determinant is -0.148176! Indeed, already in [1] the hint can be found that this inequality is necessary but not sufficient. An illustrative general proof that $|q_{12}| < q_{11}$ is necessary but not sufficient can be given with [2, fig. 4] identifying q_{11} with 2 Re (Z_{11}) and q_{12} with δ^*. In this connection, it may be emphasized finally that the most simple necessary and sufficient passivity conditions for Q are, of course, given by $q_0 \geq 0$, $q_{+1} \geq 0$, and $q_{-1} \geq 0$, applying the eigenvalues used also by Helszajn.

The above errors do not influence the further results of the Helszajn paper. However, with respect to these results some remarks may be made. These results do not replace the passivity inequalities as it is said in Section II. They give only approximation formulas for certain types of three-ports based on a lot of restrictive assumptions (e.g., the "eigennetworks" can be represented by short-circuited transmission lines, or $s_0 = -1$) which may be justified in specific cases but which are by no means true in general. For instance, (56) and (57) are valid for circulators with an equivalent circuit consisting of an ideal circulator, each port of which is loaded with a weakly lossy parallel resonant circuit, but they are not valid, e.g., for circulators with broad-band matching networks [3]. Incidentally, starting with this equivalent circuit it is interesting to prove (56) and (57) using only elementary analysis methods without reference to eigenvalue methods and transmission line theory.

In general, this should be clear: the passivity inequalities have, without further assumptions, basically an uncertainty and cannot be substituted by well-defined equations. It may be worth mentioning that meanwhile a general discussion of the passivity inequalities for the scattering matrices of cyclic-symmetric three-ports has been published [4]. No specific assumptions are made in this paper and the restrictions of the magnitudes of the scattering coefficients imposed by the passivity conditions are investigated in perfect generality. This paper is the completion of [1]. As in [1], direct relations of the magnitudes of the scattering coefficients are discussed in [4], whereas in [5] the magnitudes are related indirectly (by means of the parameter $q_{11} = 1 - |S_{11}|^2 - |S_{12}|^2 - |S_{13}|^2$), which seems to be at least somewhat impractical. The considerations in [4] culminate in necessary and sufficient conditions given by a lot of curves [4, fig. 8] which describe a body in a $|S_{11}|$, $|S_{12}|$, $|S_{13}|$-space [4, fig. 9]. Only the inside points of this body correspond to passive scattering matrices. A part of the surface of this body corresponds to the lossless case, and the projected image of this surface part yields the diagram for lossless scattering matrices [4, fig. 10], known already from [6]. However, the most important use of the results in [4] will be in the field of lossy circulators. For lossy circulators only a small part of the body under consideration is of interest. This part is illustrated by an extra diagram [4, fig. 11]. With the aid of this diagram, problems similar to the following can be solved immediately: a reflection coefficient magnitude $|S_{11}| = 0.05$ (i.e., VSWR = 1.105) may be desired; an isolation of 20 dB (i.e., $|S_{13}| = 0.1$) may be desired; furthermore, an insertion loss of 0.2 dB (i.e., $|S_{12}| = 0.9772$) may be demanded. Does this triplet satisfy the necessary and sufficient conditions for passive scattering matrices? With the aid of the diagram under consideration it is easy to see that this is not the case. (For unchanged values of $|S_{11}|$ and $|S_{13}|$ the minimum value of the insertion loss is 0.3 dB.) With respect to this example it is obvious that this diagram may be helpful in avoiding impossible requirements on circulators. Of course, semi-ideal circulators as defined in [1] are included as special cases.

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